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Standard Guide for Defining Boundary Conditions in Ground-Water Flow Modeling[1]

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1. Scope

1.1 This guide covers the specification of appropriate boundary conditions that are an essential part of conceptualizing and modeling ground-water systems. This guide describes techniques that can be used in defining boundary conditions and their appropriate application for modeling saturated ground-water flow model simulations.

1.2 This guide is one of a series of standards on ground-water flow model applications. Defining boundary conditions is a step in the design and construction of a model that is treated generally in Guide D 5447.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

1.4 *This guide offers an organized collection of information or a series of options and does not recommend a specific course of action. This document cannot replace education or experience and should be used in conjunction with professional judgment. Not all aspects of this guide may be applicable in all circumstances. This ASTM standard is not intended to represent or replace the standard of care by which the adequacy of a given professional service must be judged, nor should this document be applied without consideration of a project's many unique aspects. The word "Standard" in the title of this document means only that the document has been approved through the ASTM consensus process.*

2. Referenced Documents

2.1 *ASTM Standards:*

D 653 Terminology Relating to Soil, Rock, and Contained Fluids[2]

D 5447 Guide for Application of a Ground-Water Flow Model to a Site-Specific Problem[3]

3. Terminology

3.1 *Definitions:*

3.1.1 *aquifer, confined*—an aquifer bounded above and below by confining beds and in which the static head is above the top of the aquifer.

3.1.2 *boundary*—geometrical configuration of the surface enclosing the model domain.

3.1.3 *boundary condition*—a mathematical expression of the state of the physical system that constrains the equations of the mathematical model.

3.1.4 *conceptual model*—a simplified representation of the hydrogeologic setting and the response of the flow system to stress.

3.1.5 *flux*—the volume of fluid crossing a unit cross-sectional surface area per unit time.

3.1.6 *ground-water flow model*—an application of a mathematical model to the solution of a ground-water flow problem.

3.1.7 *hydraulic conductivity—(field aquifer tests),* the volume of water at the existing kinematic viscosity that will move in a unit time under unit hydraulic gradient through a unit area measured at right angles to the direction of flow.

3.1.8 *hydrologic condition*—a set of ground-water inflows or outflows, boundary conditions, and hydraulic properties that cause potentiometric heads to adopt a distinct pattern.

3.1.9 *simulation*—one complete execution of the computer program, including input and output.

3.1.10 *transmissivity*—the volume of water at the existing kinematic viscosity that will move in a unit time under a unit hydraulic gradient through a unit width of the aquifer.

3.1.11 *unconfined aquifer*—an aquifer that has a water table.

3.1.12 For definitions of other terms used in this test method, see Terminology D 653.

4. Significance and Use

4.1 Accurate definition of boundary conditions is an essential part of conceptualizing and modeling ground-water flow systems. This guide describes the properties of the most common boundary conditions encountered in ground-water systems and discusses major aspects of their definition and application in ground-water models. It also discusses the significance and specification of boundary conditions for some

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[2] *Annual Book of ASTM Standards*, Vol 04.08.

[3] *Annual Book of ASTM Standards*, Vol 04.09.

field situations and some common errors in specifying boundary conditions in ground-water models.

5. Types of Boundaries

5.1 The flow of ground water is described in the general case by partial differential equations. Quantitative modeling of a ground-water system entails the solution of those equations subject to site-specific boundary conditions.

5.2 *Types of Modeled Boundary Conditions*—Flow model boundary conditions can be classified as specified head or Dirichlet, specified flux or Neumann, a combination of specified head and flux, or Cauchy, free surface boundary, and seepage-face. Each of these types of boundaries and some of their variations are discussed below.

5.2.1 *Specified Head, or Dirichlet, Boundary Type*—A specified head boundary is one in which the head can be specified as a function of position and time over a part of the boundary surface of the ground-water system. A boundary of specified head may be the general type of specified head boundary in which the head may vary with time or position over the surface of the boundary, or both, or the constant-head boundary in which the head is constant in time, but head may differ in position, over the surface of the boundary. These two types of specified head boundaries are discussed below.

5.2.1.1 *General Specified-Head Boundary*—The general type of specified-head boundary condition occurs wherever head can be specified as a function of position and time over a part of the boundary surface of a ground-water system. An example of the simplest type might be an aquifer that is exposed along the bottom of a large stream whose stage is independent of ground-water seepage. As one moves upstream or downstream, the head changes in relation to the slope of the stream channel and the head varies with time as a function of stream flow. Heads along the stream bed are specified according to circumstances external to the ground-water system and maintain these specified values throughout the problem solution, regardless of changes within the ground-water system.

5.2.1.2 *Constant-Head Boundary*—A constant head boundary is boundary in which the aquifer system coincides with a surface of unchanging head through time. An example is an aquifer that is bordered by a lake in which the surface-water stage is constant over all points of the boundary in time and position or an aquifer that is bordered by a stream of constant flow that is unchanging in head with time but differs in head with position.

5.2.2 *Specified Flux or Neumann Boundary Type*—A specified flux boundary is one for which the flux across the boundary surface can be specified as a function of position and time. In the simplest type of specified-flux boundary, the flux across a given part of the boundary surface is considered uniform in space and constant with time. In a more general case, the flux might be constant with time but specified as a function of position. In the most general case, flux is specified as a function of time as well as position. In all cases of specified flux boundaries, the flux is specified according to circumstances external to the ground-water flow system and the specified flux values are maintained throughout the problem solution regardless of changes within the ground-water flow system.

5.2.2.1 *No Flow or Streamline Boundary*—The no-flow or streamline boundary is a special case of the specified flux boundary. A streamline is a curve that is tangent to the flow-velocity vector at every point along its length; thus no flow crosses a streamline. An example of a no-flow boundary is an impermeable boundary. Natural earth materials are never impermeable. However, they may sometimes be regarded as effectively impermeable for modeling purposes if the hydraulic conductivities of the adjacent materials differ by orders of magnitude. Ground-water divides are normal to streamlines and are also no-flow boundaries. However, the ground-water divide does not intrinsically correspond to physical or hydraulic properties of the aquifer. The position of a ground-water divide is a function of the response of the aquifer system to hydrologic conditions and may be subject to change with changing conditions. The use of ground-water divides as model boundaries may produce invalid results.

5.2.3 *Head Dependent Flux, or Cauchy Type*—In some situations, flux across a part of the boundary surface changes in response to changes in head within the aquifer adjacent to the boundary. In these situations, the flux is a specified function of that head and varies during problem solution as the head varies.

NOTE 1—An example of this type of boundary is the upper surface of an aquifer overlain by a confining bed that is in turn overlain by a body of surface water. In this example, as in most head-dependent boundary situations, a practical limit exists beyond which changes in head cease to cause a change in flux. In this example, the limit will be reached where the head within the aquifer falls below the top of the aquifer so that the aquifer is no longer confined at that point, but is under an unconfined or water-table condition, while the confining bed above remains saturated. Under these conditions, the bottom of the confining bed becomes locally a seepage face. Thus as the head in the aquifer is drawn down further, the hydraulic gradient does not increase and the flux through the confining bed remains constant. In this hypothetical case, the flux through the confining bed increases linearly as the head in the aquifer declines until the head reaches the level of the base of the confining bed after which the flux remains constant. Another example of a head dependent boundary with a similar behavior is evapotranspiration from the water table, where the flux from the water table is often modeled as decreasing linearly with depth to water and becomes zero where the water table reaches some specified “cutoff” depth.

5.2.4 *Free-Surface Boundary Type*—A free-surface boundary is a moveable boundary where the head is equal to the elevation of the boundary. The most common free-surface boundary is the water table, which is the boundary surface between the saturated flow field and the atmosphere (capillary zone not considered). An important characteristic of this boundary is that its position is not fixed; that is its position may rise and fall with time. In some problems, for example, flow through an earth dam, the position of the free surface is not known before but must be found as part of the problem solution.

5.2.4.1 Another example of a free surface boundary is the transition between freshwater and underlying seawater in a coastal aquifer. If diffusion is neglected and the salty ground water seaward of the interface is assumed to be static, the freshwater-saltwater transition zone can be treated as a sharp interface and can be taken as the bounding stream surface (no-flow) boundary of the fresh ground-water flow system. Under these conditions, the freshwater head at points on the interface varies only with the elevation and the freshwater head at any point on this idealized stream-surface boundary is thus a linear function of the elevation head of that point.

5.2.5 *Seepage-Face Boundary Type*—A surface of seepage is a boundary between the saturated flow field and the atmosphere along which ground water discharges, either by evaporation or movement "downhill" along the land surface as a thin film in response to the force of gravity. The location of this type of boundary is generally fixed, but its length is dependent upon other system boundaries. A seepage surface is always associated with a free surface boundary. Seepage faces are commonly neglected in models of large aquifer systems because their effect is often insignificant at a regional scale of problem definition. However, in problems defined over a smaller area, which require more accurate system definition, they must be considered.

6. Procedure

6.1 The definition of boundary conditions of a model is a part of the application of a model to a site-specific problem (see Guide D 5447). The steps in boundary definition may be stated as follows:

6.1.1 Identification of the physical boundaries of the flow system boundaries,

6.1.2 Formulation of the mathematical representation of the boundaries,

6.1.3 Examination and sensitivity testing of boundary conditions that change when the system is under stress, that is, stress-dependent boundaries, and

6.1.4 Revision and final formulation of the initial model boundary representation.

6.1.5 Further examination, testing, and refinement of the model boundaries is a part of the verification and validation process of the application of each model and is discussed in Guide D 5447.

6.2 *Boundary Identification*—Identify as accurately as possible the physical boundaries of the flow system. The three-dimensional bounding surfaces of the flow system must be defined even if the model is to be represented by a two-dimensional model. Even if the lateral boundaries are distant from the region of primary interest, it is important to understand the location and hydraulic conditions on the boundaries of the flow system.

6.2.1 *Ground-Water Divides*—Ground-water divides have been chosen as boundaries by some modelers because they can be described as stream lines and can be considered as no flow boundaries. However, the locations of ground-water divides depend upon hydrologic conditions in the sense that they can move or disappear in response to stress on the system. For these reasons, ground-water divides are not physical boundaries of the flow system.[4] Their representation as no-flow boundaries can sometimes be justified if the objective of the simulation is to gain an understanding of natural flow without applied stress or if the changed conditions used for simulation can be shown, for example, by sensitivity analysis, to have a negligible effect on the position of the boundary.

6.2.2 *Water Table*—The water table is an important boundary in many ground-water flow systems and various ways of treating the water table may be appropriate in different ground-water models. The position of the water table is not fixed and the water table boundary may act as a source or sink of water. Some of these ways of treating the water table are discussed below.

6.2.2.1 The position of the water table is not fixed, but it may be appropriate to treat the water table as a constant-head boundary in a steady-state simulation where the flow distribution in an unstressed model is simulated.

6.2.2.2 The water table may be represented as a free-surface boundary with recharge, in which case, the water table is neither a potential nor a stream surface.

6.2.2.3 The water table may be represented as a free surface boundary with discharge in which discharge is by evapotranspiration as a function of depth to water. The boundary in this case is a head-dependent flux boundary.

6.2.2.4 A sloping water table may be represented as a flow surface, that is, a locus of flow lines, where accretion is zero.

6.2.2.5 The water table may be a surface at which accretion, the net rate of gain or loss normal to the aquifer surface, is a function of time and location.

6.3 *Model Representation*—Formulate the model representation for the bounding surfaces of the flow system. Define the hydraulic conditions on the boundaries: specified head, specified flux, head-dependent flux, free surface boundary or seepage face.

6.4 *Stress Dependency*—Examine the stress-dependence of each boundary. Perform sensitivity analysis of boundaries to determine their stress dependency and to determine if natural boundaries are compatible with the representation in the model.

6.4.1 For example, a specified head boundary assumes the head is independent of the stress in the model. If the stress applied to the real system will affect the head on the boundary, the boundary is stress-dependent and modeling the boundary as a specified head boundary is not a valid representation of the boundary. Likewise, specified flux boundaries assume the flux to or from the model is independent of the stress in the model and if flux to or from the model is dependent upon head in the model, the boundary is a stress-dependent boundary and requires such recognition in representing the boundary.

6.4.1.1 Consider the physical boundary in relation to system stress to be applied during simulation. The model representation of a system boundary may be a function of the nature and magnitude of stress applied to the system during model simulation. Consider, for example, a small to medium-sized stream, which may function as a specified head boundary if the stress does not induce flow to or from the stream of sufficient magnitude to significantly affect the stream stage. If, however, the stress is so large as to cause a part of the stream to dry up, then the stream can no longer be treated as a specified head boundary. The stream may need to be modeled as a flux dependent head boundary.

6.4.1.2 If the boundary conditions are stress dependent, the

[4] Franke, O. L., Reilly, T. E., and Bennett, G. D., "Definition of Boundary and Initial Conditions in the Analysis of Ground-Water Flow Systems—An Introduction," *Techniques of Water-Resources Investigations of the United States Geological Survey, Book 3, Chapter B5, 1987.*

model cannot be considered a general, all-purpose tool for investigating any stress on the system because it will give valid results only when the stresses do not impact the boundary. The study of a new stress on the same model may require the reformulation of the representation of boundaries of the model and sensitivity tests on the model boundary representation.

6.4.1.3 Stress-dependency is of primary concern wherever the model boundaries differ from the natural system boundaries. For example, model boundaries that may differ from physical boundaries of the flow system include natural boundaries that may extend beyond the boundaries of the model. Prepare a careful justification to show that the proposed boundary is appropriate and will not cause the model solution to differ substantially from the response that would occur in the real system.

6.5 The results of stress-dependency tests should be documented with regard to stress conditions and the magnitude of impact on stress-dependent boundaries.

6.6 *Revise Model Boundary Representation*—Based on the sensitivity testing, revise model boundary representations and document the ranges of stress for which the boundaries are designed.

7. Report

7.1 Completely document the boundary definition of the models. Such documentation will be a part of the overall documentation of the model. Include the following items pertaining to the formulation of model boundaries in the model report:

7.1.1 Describe the natural physical boundaries of the model and the processes operating at the boundaries, and

7.1.2 Describe the formulation of the model boundaries, the stress dependency of the boundaries and the model representation of each boundary. Evaluate the sensitivity analysis of the boundaries and state the conditions of stress over which the modeled boundary conditions are appropriate.

8. Keywords

8.1 aquifers; boundary condition; ground-water model